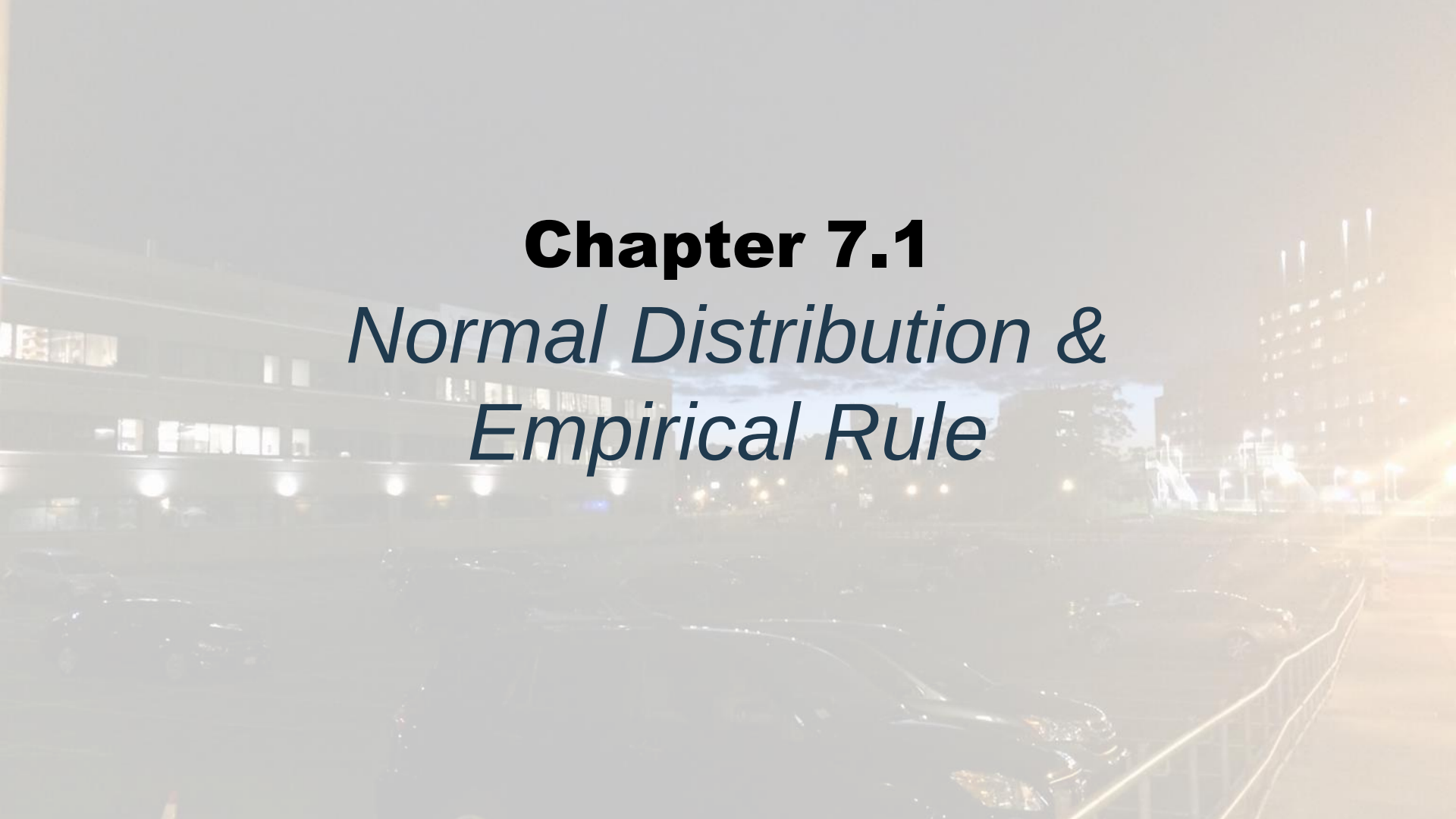


Chapter 7.1

Normal Distribution & Empirical Rule

Learning Objectives

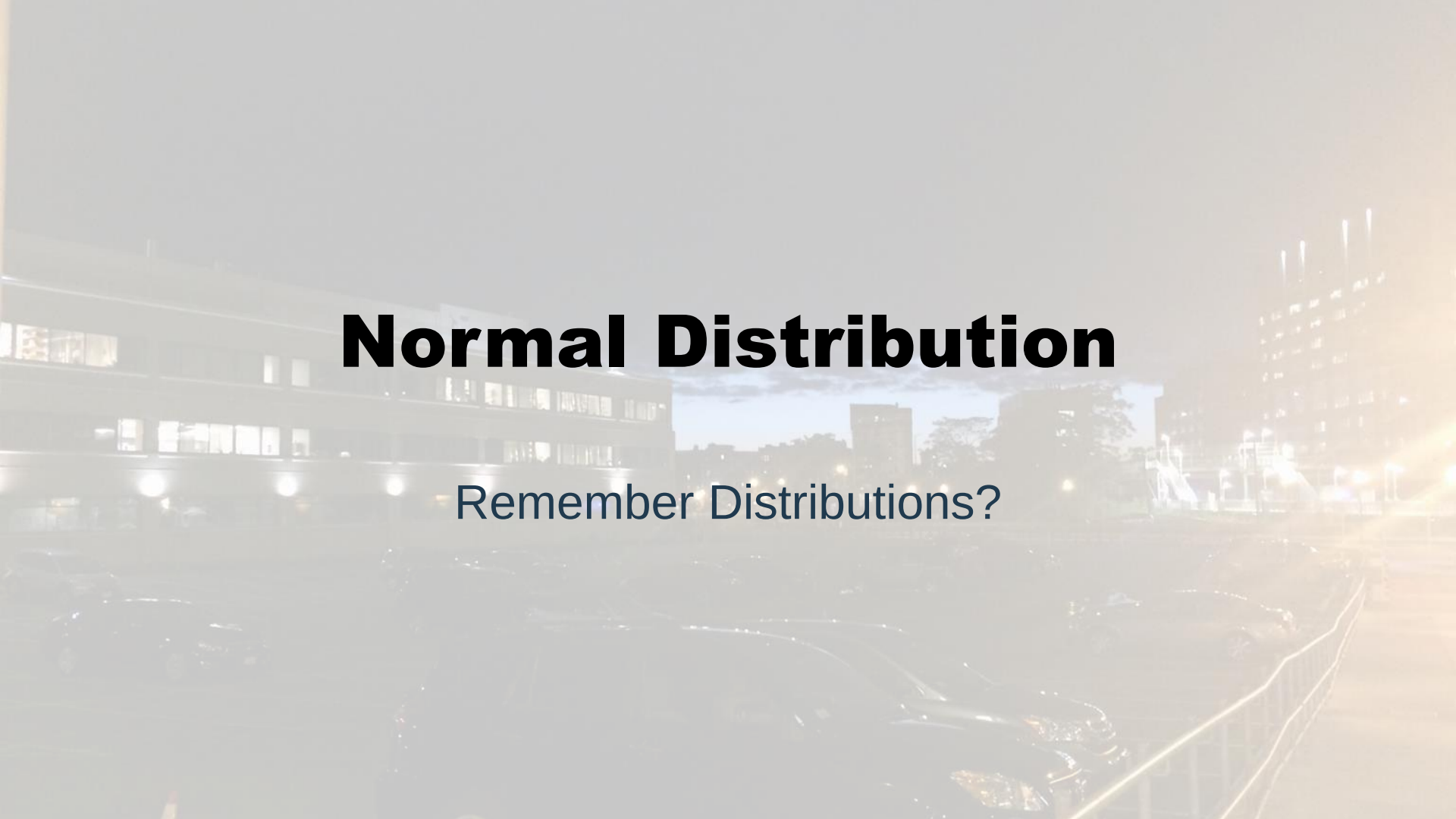
At the end of this lecture, the student should be able to:

- State two properties of the normal curve.
- State two differences between Chebyshev Intervals and the Empirical Rule
- Explain how to apply the Empirical Rule to a normal distribution

Introduction

- Remembering distributions
- Properties of the normal curve
- Remembering Chebyshev Intervals
- Empirical Rule
- Applying the Empirical Rule

Photograph by Mightyhansa



Normal Distribution

Remember Distributions?

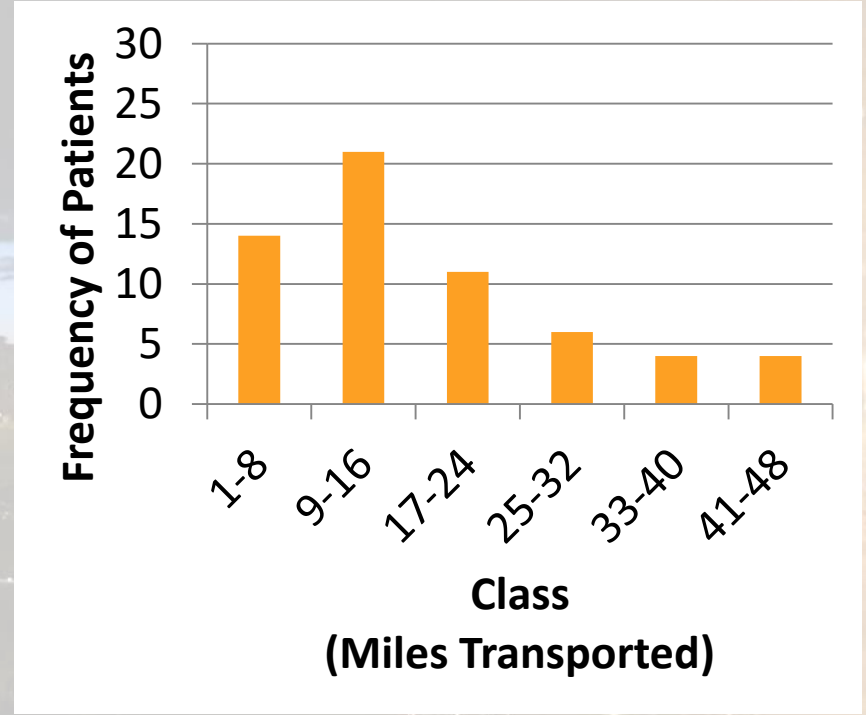
Remember Distributions?

- Using quantitative variable
- Classes determined
- Frequency table made

Class Limits	Freq-uency	Relative Freq-uency
1-8 miles	14	0.23
9-16 miles	21	0.35
17-24 miles	11	0.18
25-32 miles	6	0.10
33-40 miles	4	0.07
41-48 miles	4	0.07
Total	60	1.00

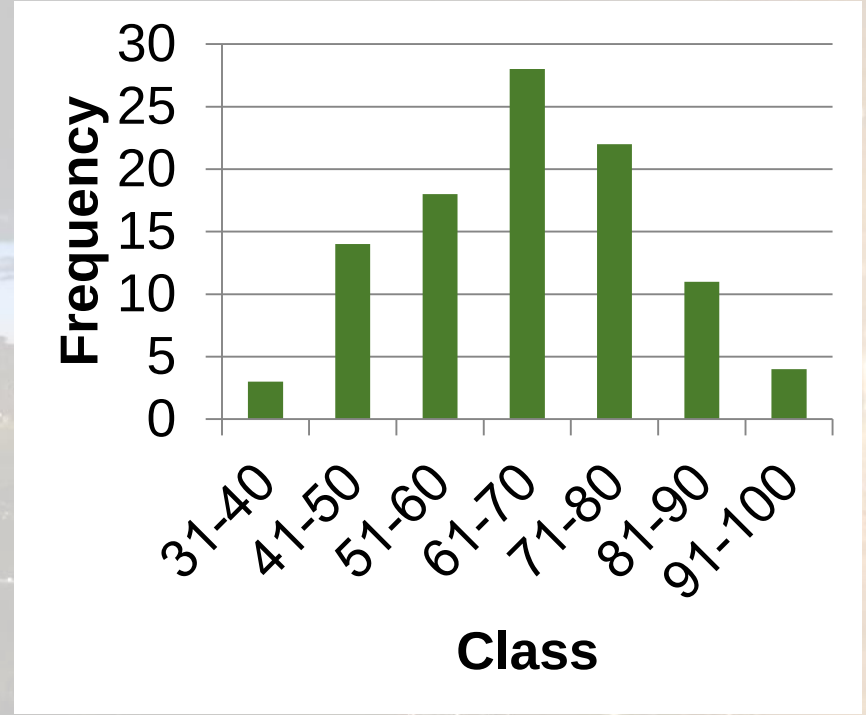
Remember Distributions?

- Using quantitative variable
- Classes determined
- Frequency table made
- Frequency histogram
- Then we could see the distribution



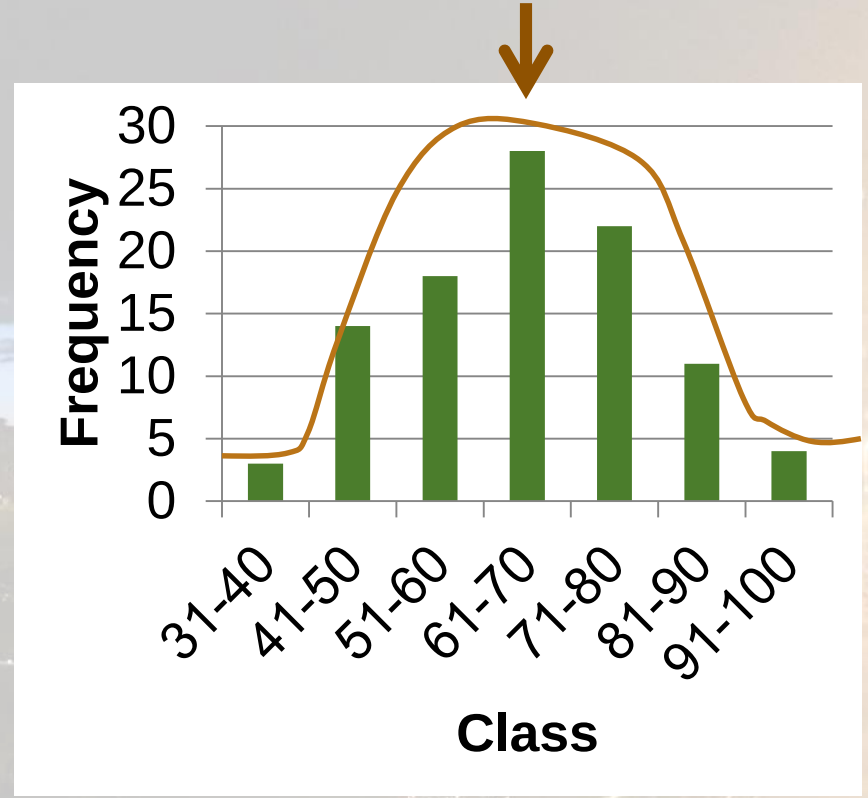
Remember the Normal Distribution?

- Imagine a large class (n=100) takes a very difficult test
- The test is worth 100 points, but it's so hard, no one actually gets 100 points
- Instead, the mode is near a C grade



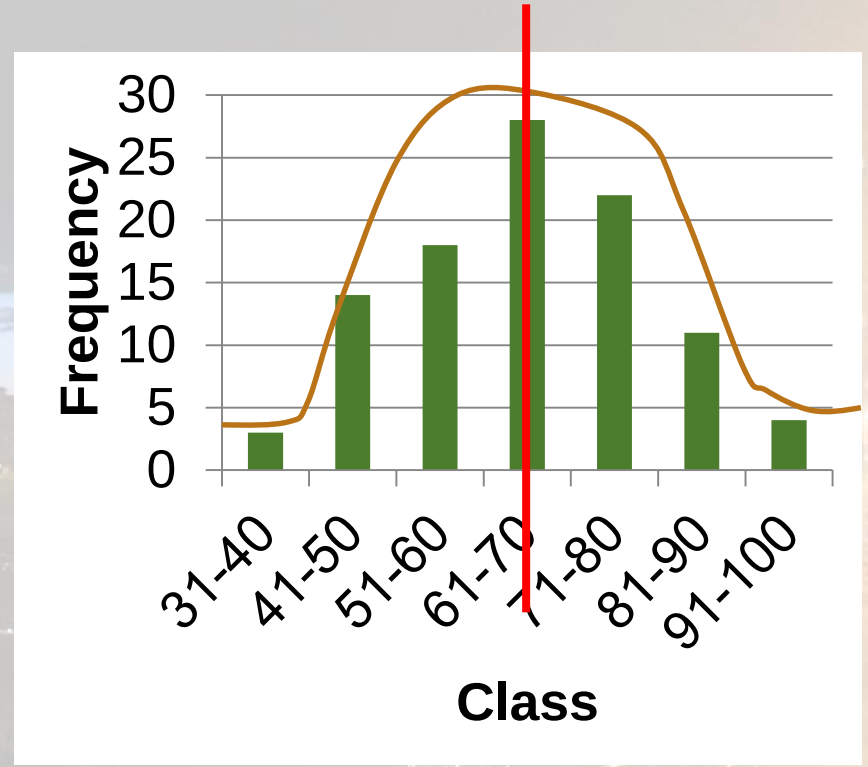
Properties of the Normal Curve

1. The curve is bell-shaped, with the highest point over the mean.



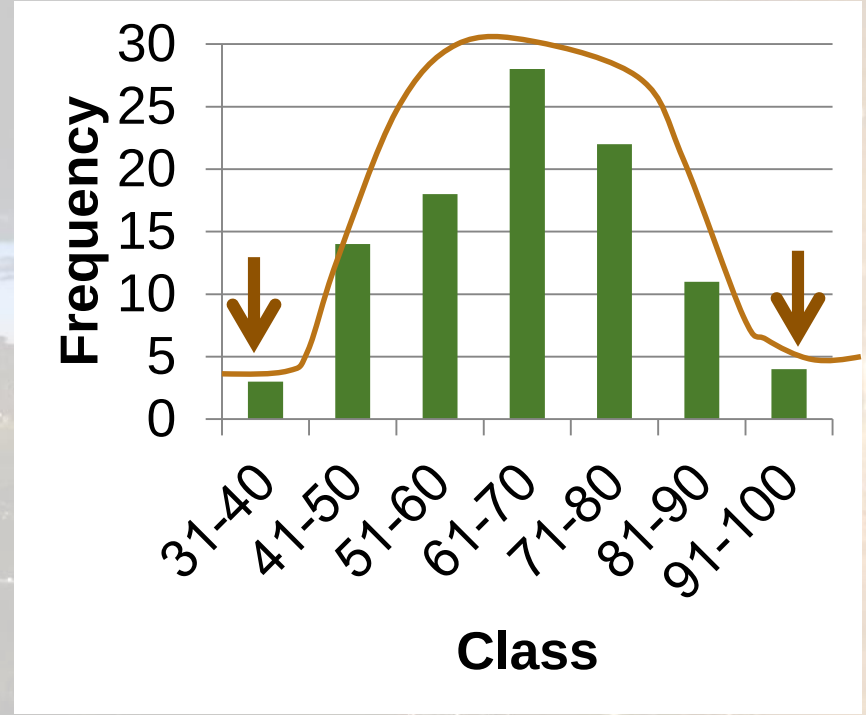
Properties of the Normal Curve

1. The curve is bell-shaped, with the highest point over the mean.
2. The curve is symmetrical around a vertical line through the mean.



Properties of the Normal Curve

1. The curve is bell-shaped, with the highest point over the mean.
2. The curve is symmetrical around a vertical line through the mean.
3. The curve approaches the horizontal axis but never touches or crosses it.



Properties of the Normal Curve

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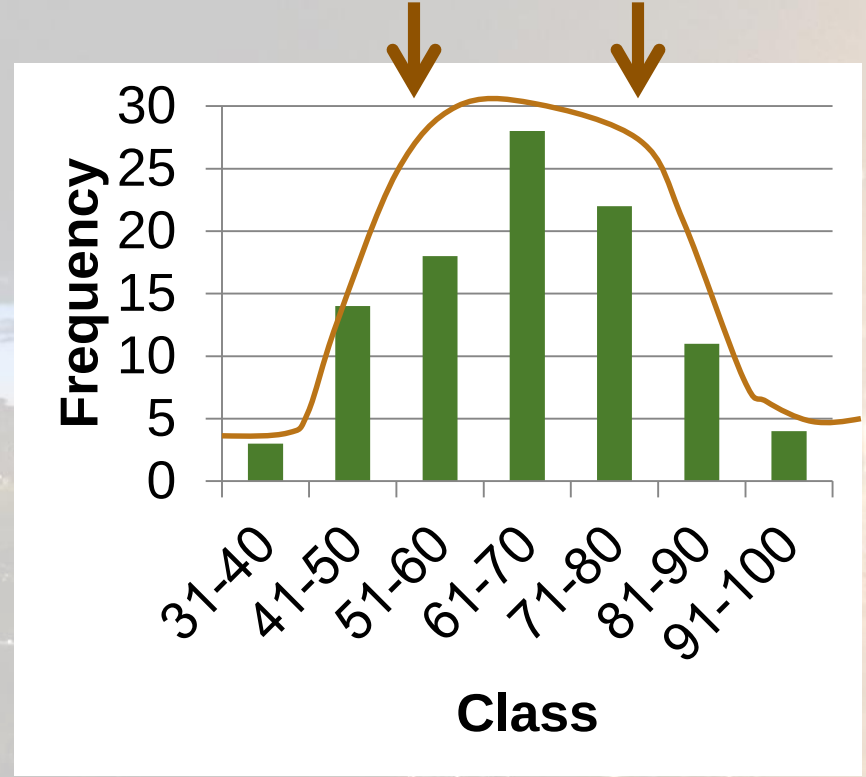
3. The curve approaches the horizontal axis but never touches or crosses it.
4. The inflection (transition) points between cupping upward and downward occur at about mean +/- 1 sd



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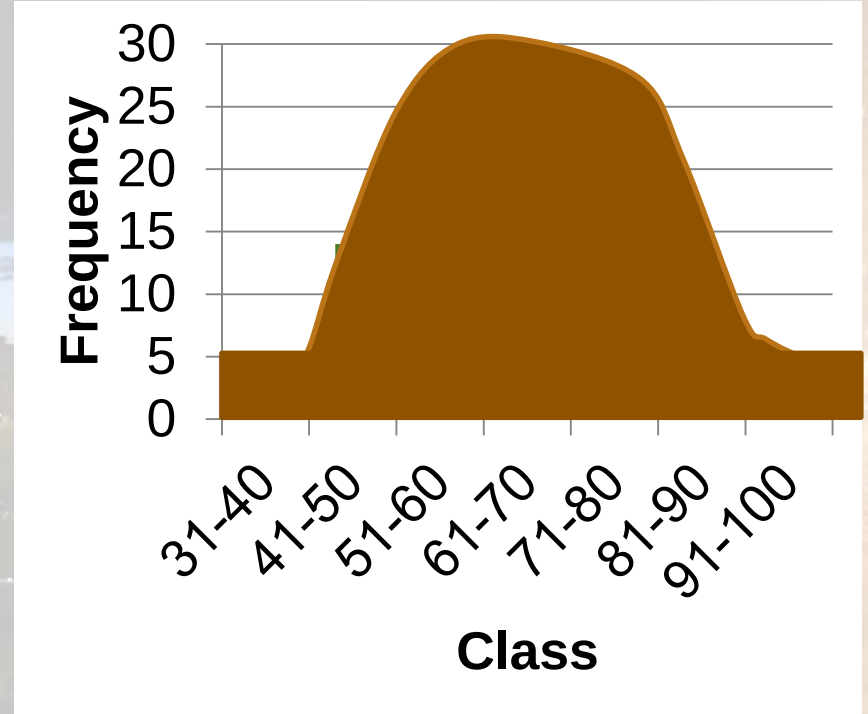
5. The area under the entire curve is 1 (think: 100%).



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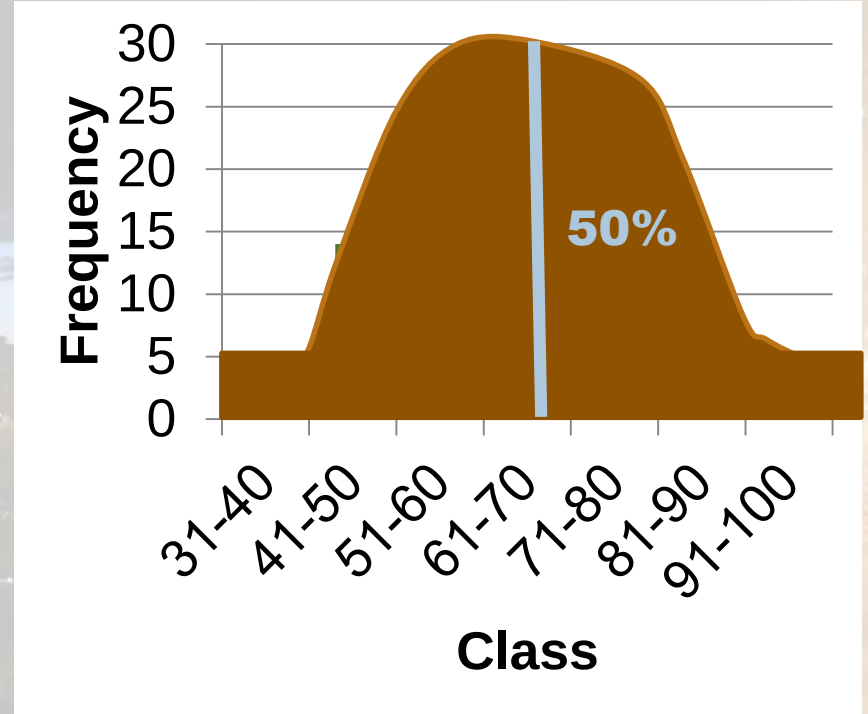

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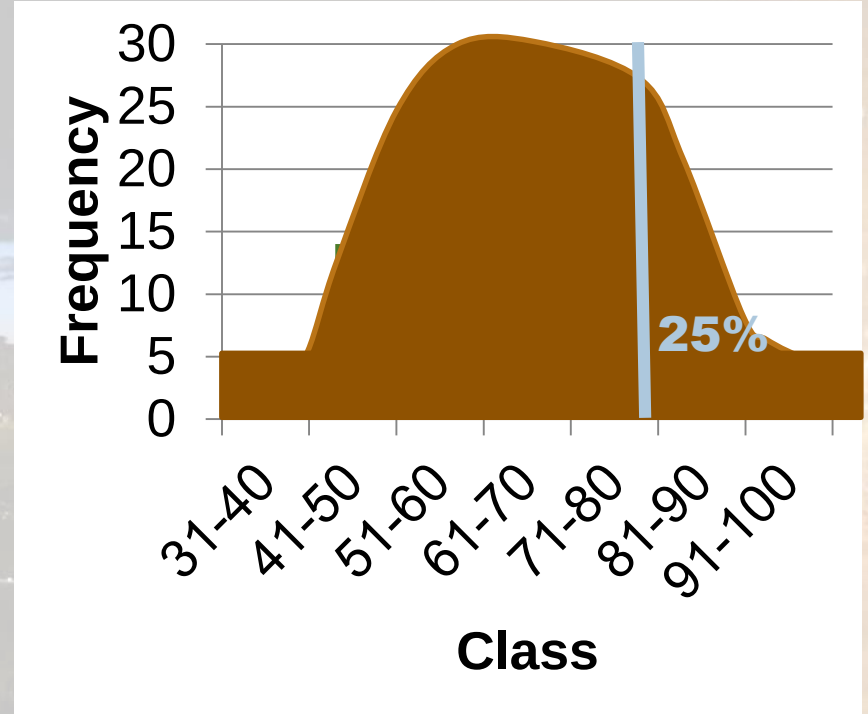


Empirical Rule

Remember Chebyshev?

Remember Chebyshev?

- Intervals have boundaries, or limits: lower limit and upper limit.
- Remember Chebyshev Intervals?
  - They say, “At least ____% of the data fall in the interval.”
  - When lower limit was μ-2σ, and upper limit was μ+2σ, at least 75% of the data were in the interval.
- Imagine n=100 students, μ score on test 65.5, σ = 14.5
  - Lower limit: 65.5 – (2*14.5) = 36.5
  - Upper limit: 65.5 + (2*14.5) = 94.5
  - So if you had 100 data points, *at least* 75 would be between 36.5 and 94.5.

Chebyshev vs. Empirical Rule

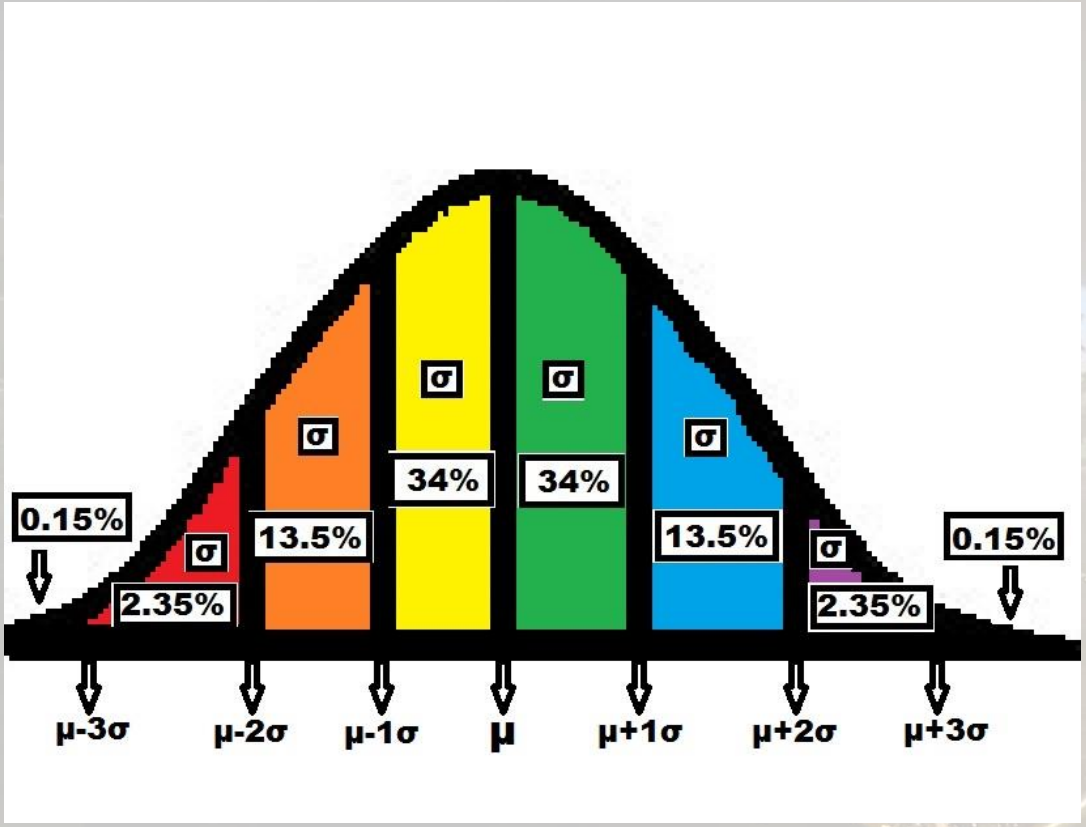
Chebyshev's Theorem

1. Applies to any distribution
2. Says "at least"
   - between μ +/- 2σ, there are *AT LEAST* 75% of the data
   - between μ +/- 3σ is at least 88.9%
   - between μ +/- 4σ is at least 93.8%

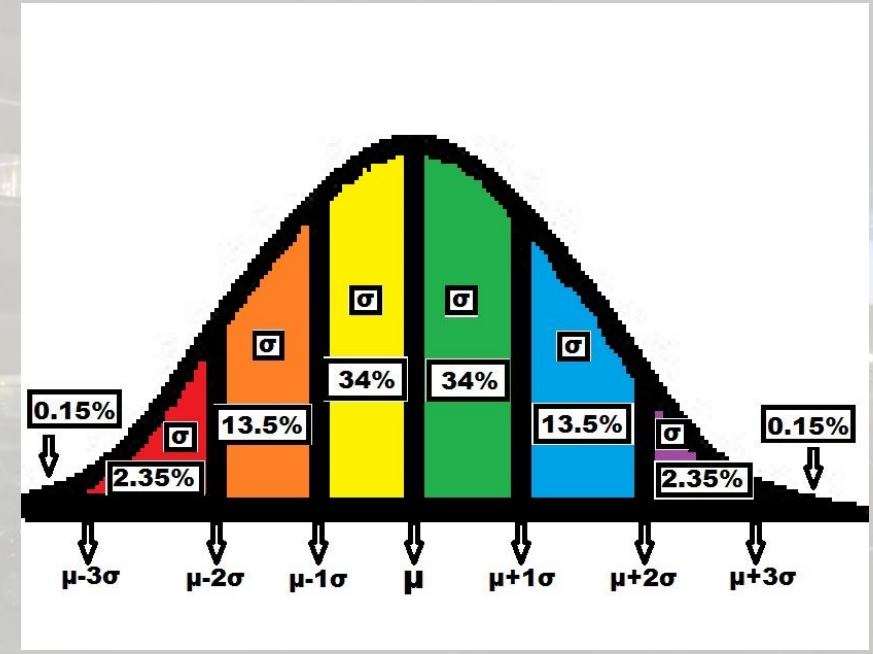
Empirical Rule

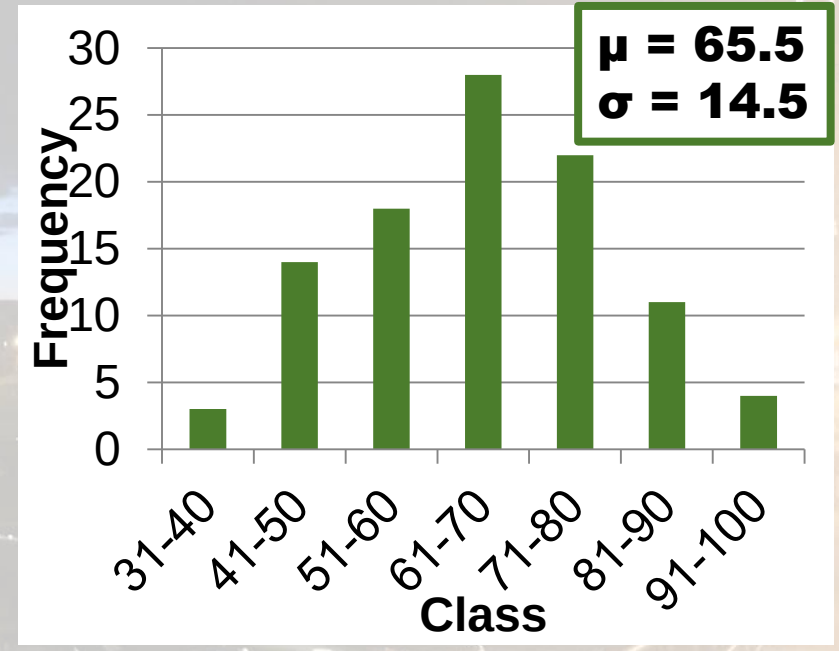
1. Applies to *ONLY* the normal distribution
2. Says "approximately"
   - 68% of the data are in interval μ +/- 1σ
   - 95% in interval μ +/- 2σ
   - 99.7% (almost all) in interval μ +/- 3σ

Empirical Rule Diagram

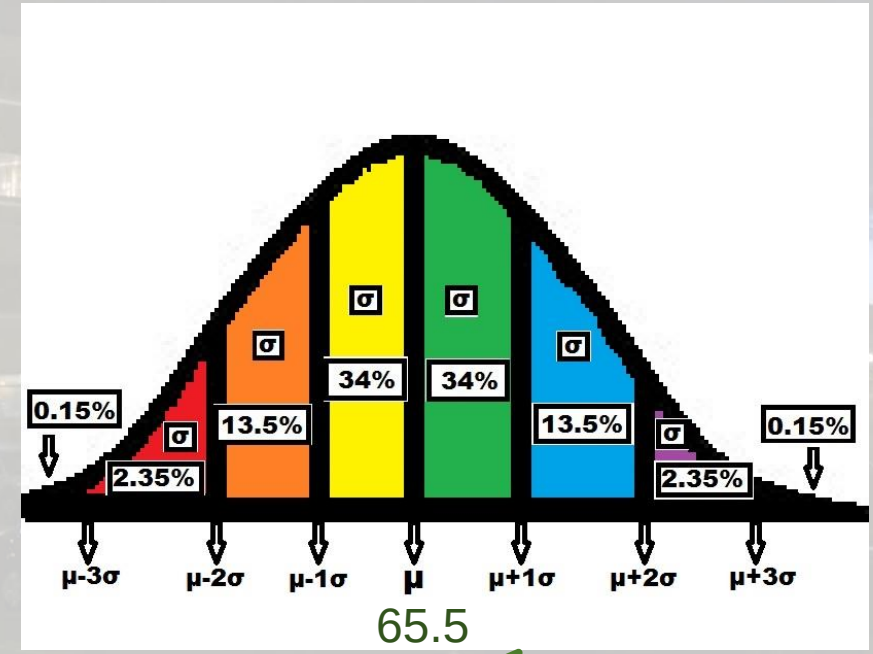


Applying Empirical Rule

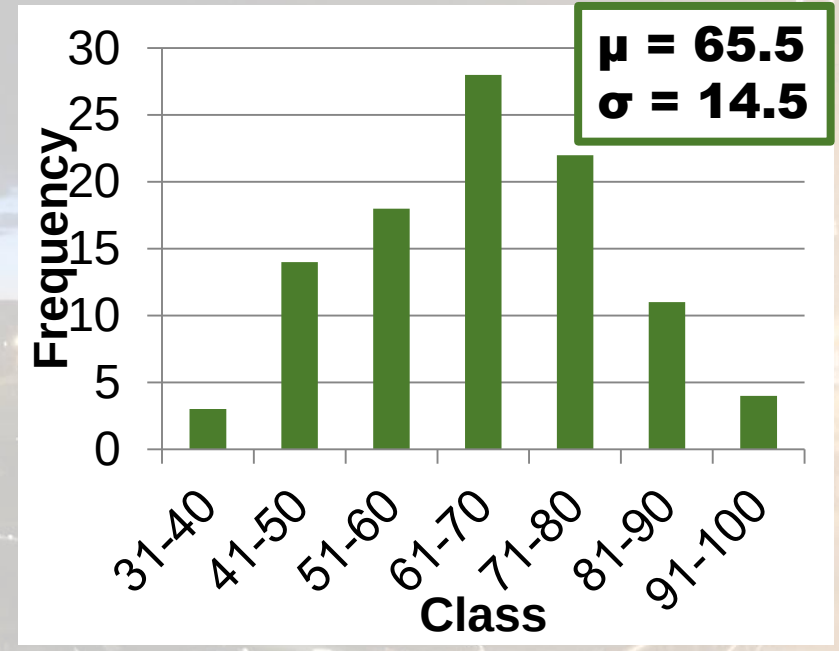




Applying Empirical Rule

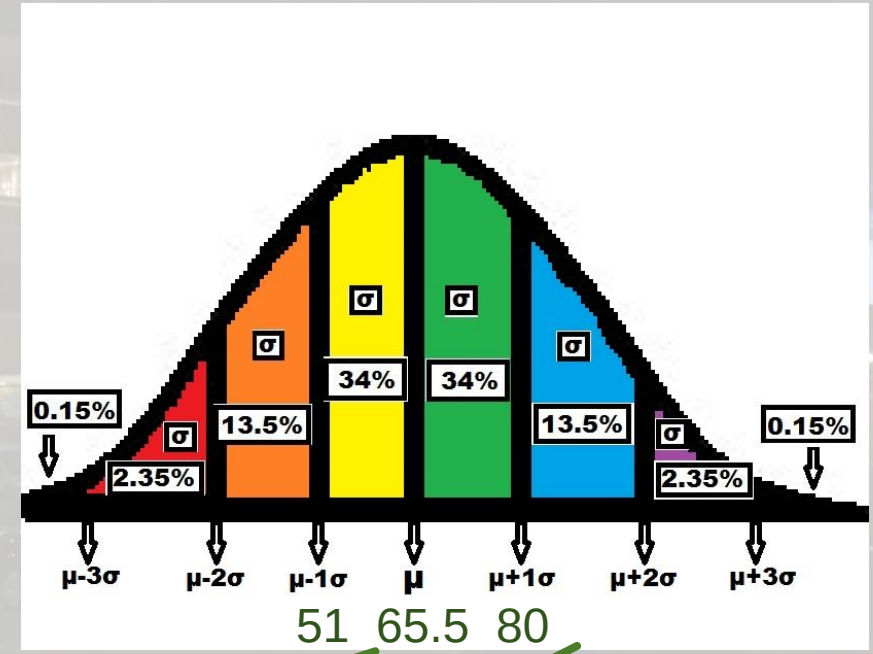


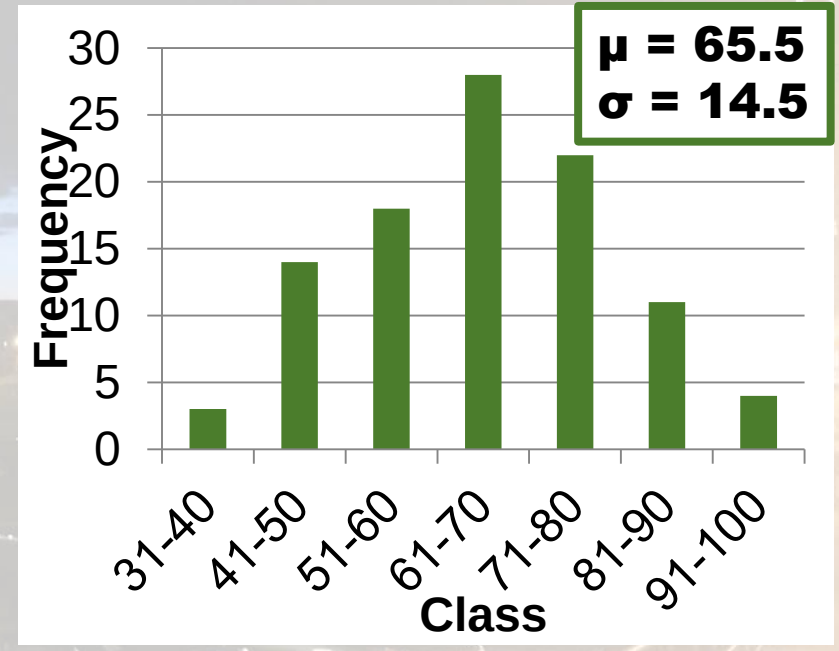
65.5



We can add the μ.

Applying Empirical Rule

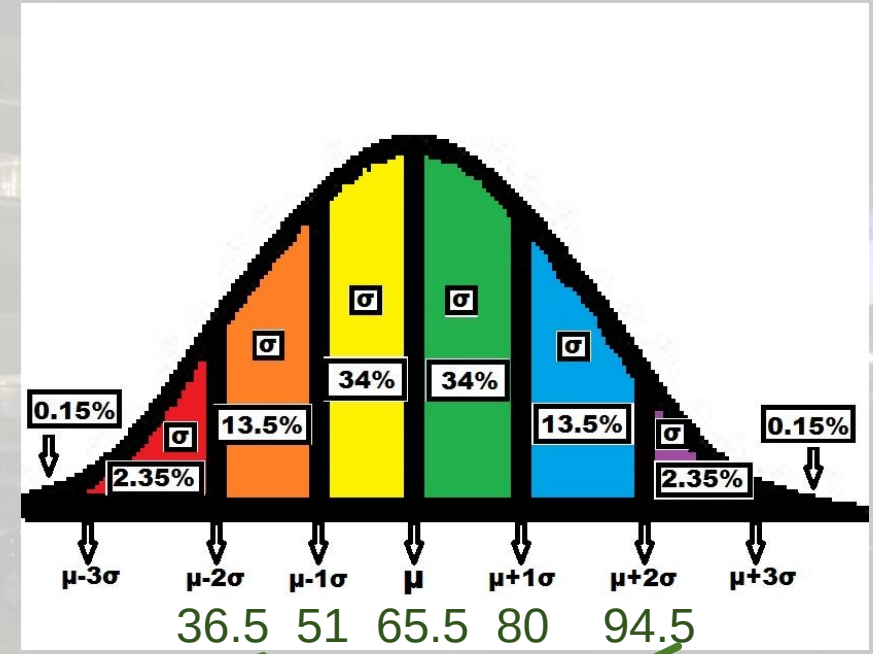


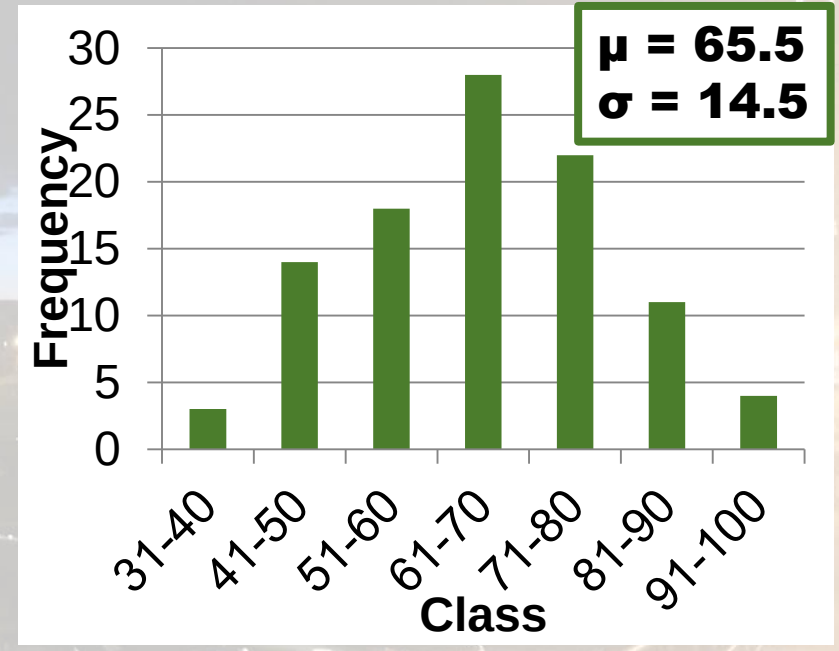


65.5 – (1*14.5) = 51

65.5 + (1*14.5) = 80

Applying Empirical Rule

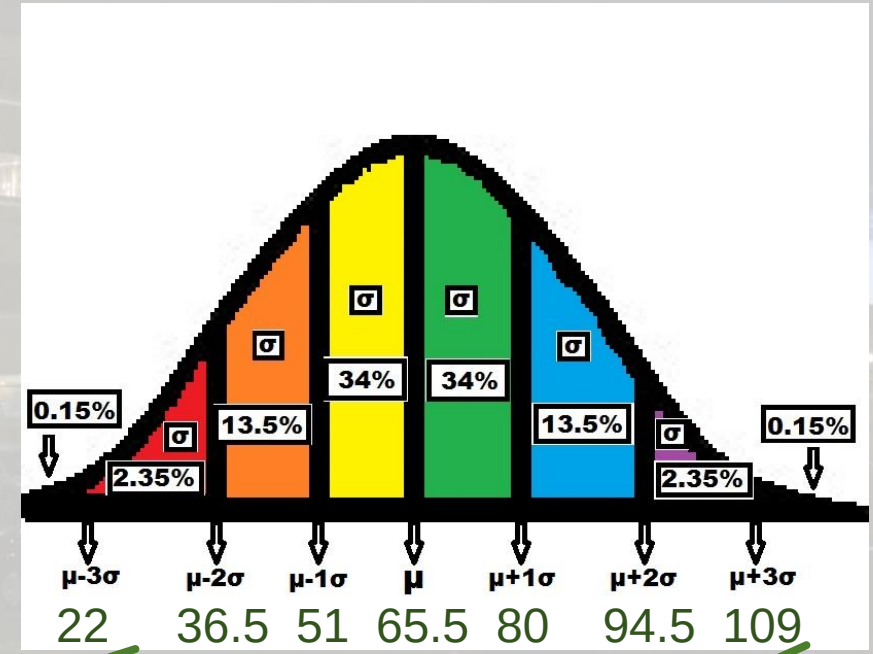


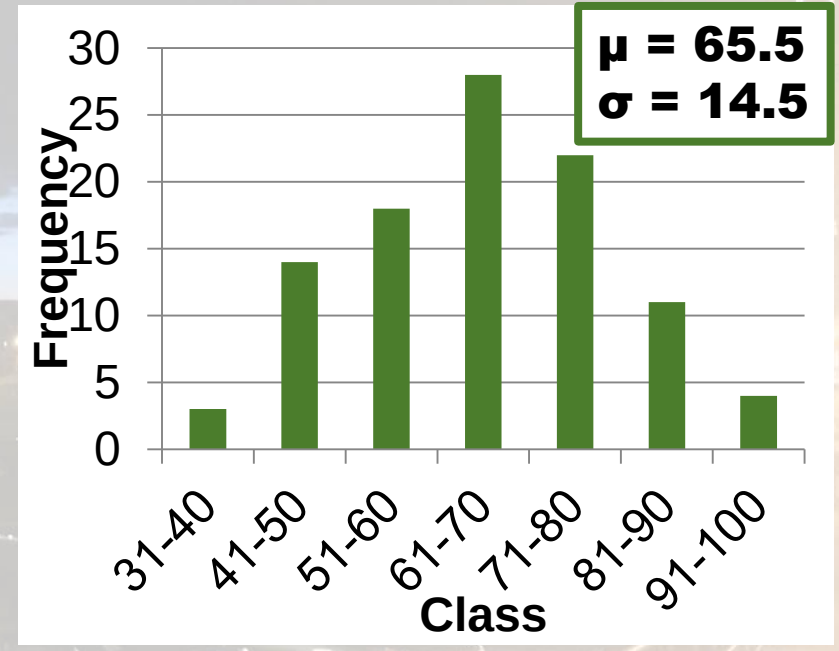


65.5 – (2*14.5) = 36.5

65.5 + (2*14.5) = 94.5

Applying Empirical Rule

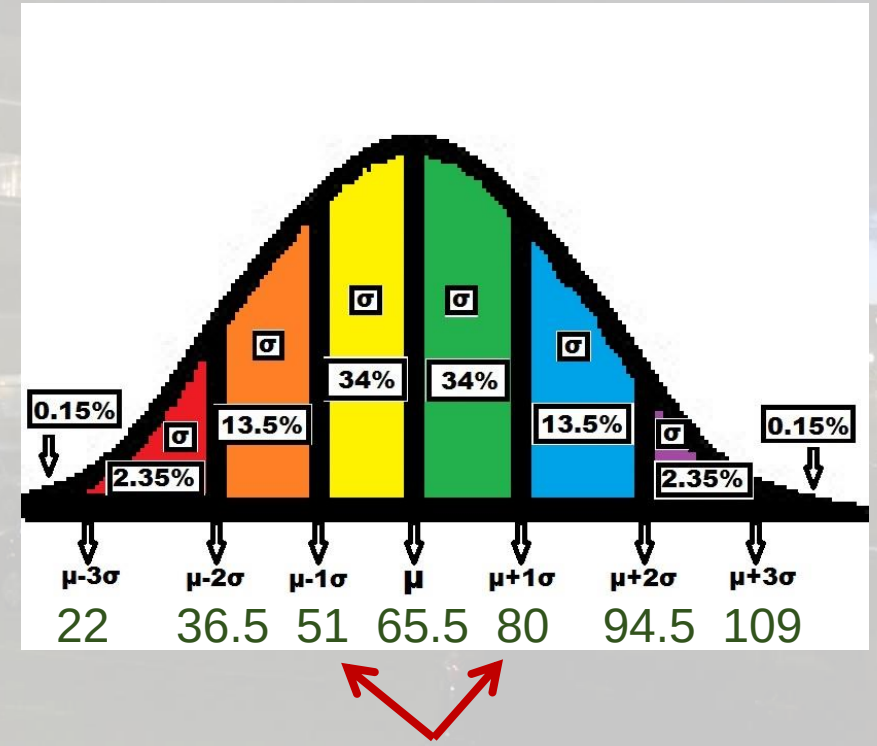




65.5 – (3*14.5) = 22

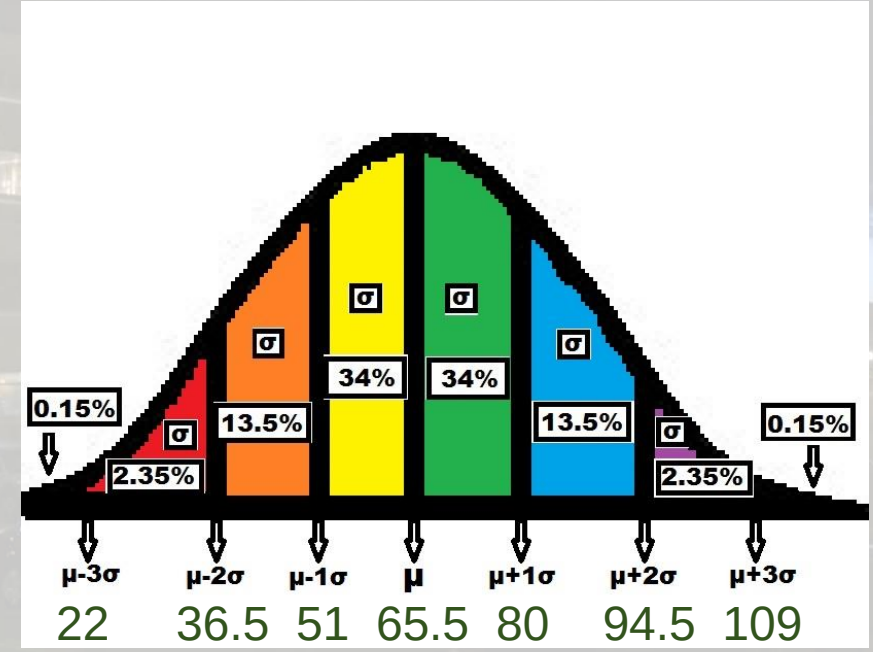
65.5 + (3*14.5) = 109

Applying Empirical Rule



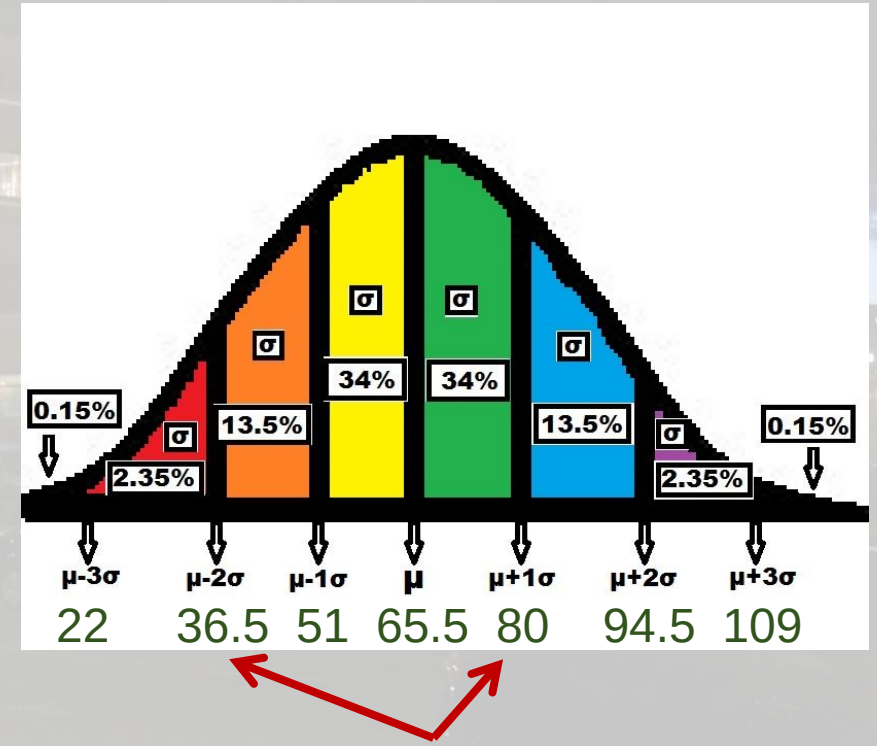
- Remember n=100?
- 34% of scores are between 51 and 65.5, meaning 34 scores.
- Another 34% (n=34) are between 65.5 and 80.
- This means 34%+34%=68% (n=68) are between 51 and 80.

Applying Empirical Rule



Question: What % of the data (student scores) are between 36.5 and 80?

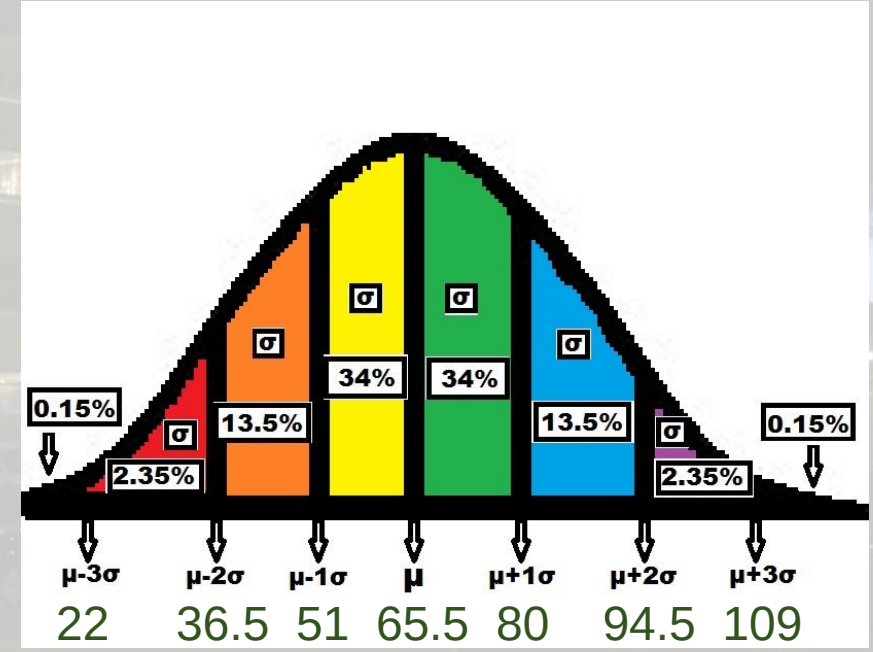
Applying Empirical Rule



Question: What % of the data (student scores) are between 36.5 and 80?

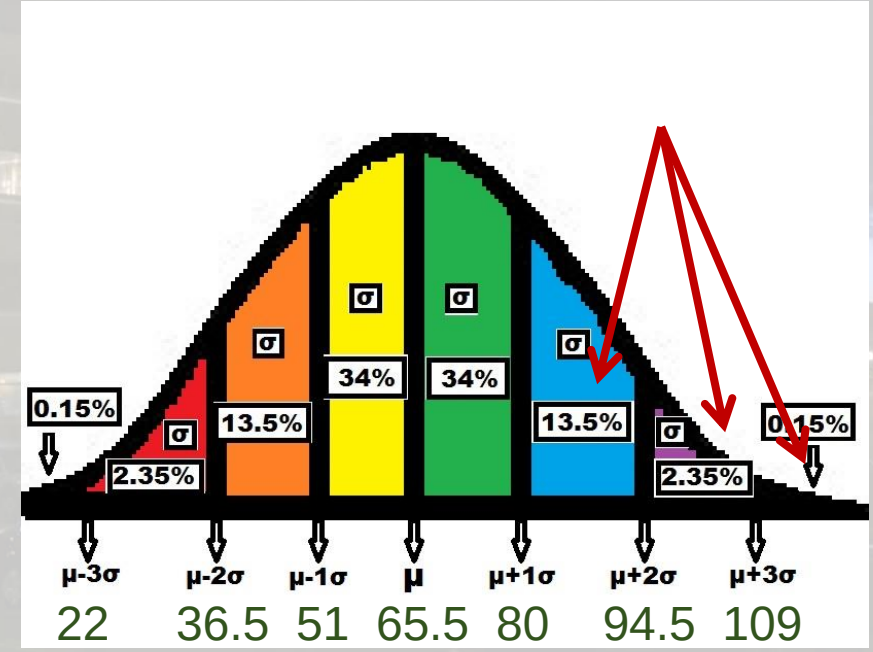
Answer: 13.5% + 34% + 34% = 81.5%

Applying Empirical Rule



Question: What cutpoint marks the top 16% of scores?

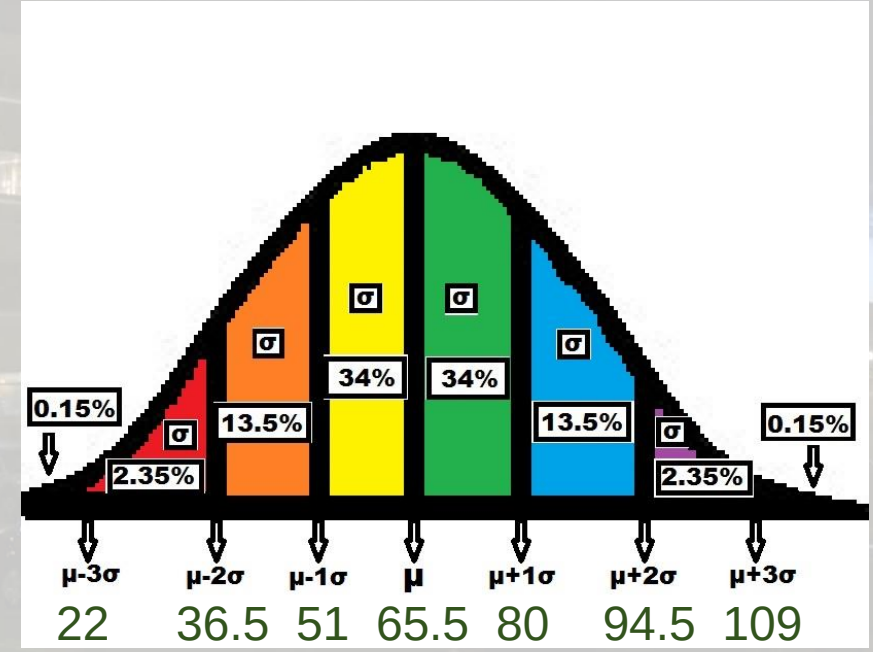
Applying Empirical Rule



Question: What cutpoint marks the top 16% of scores?

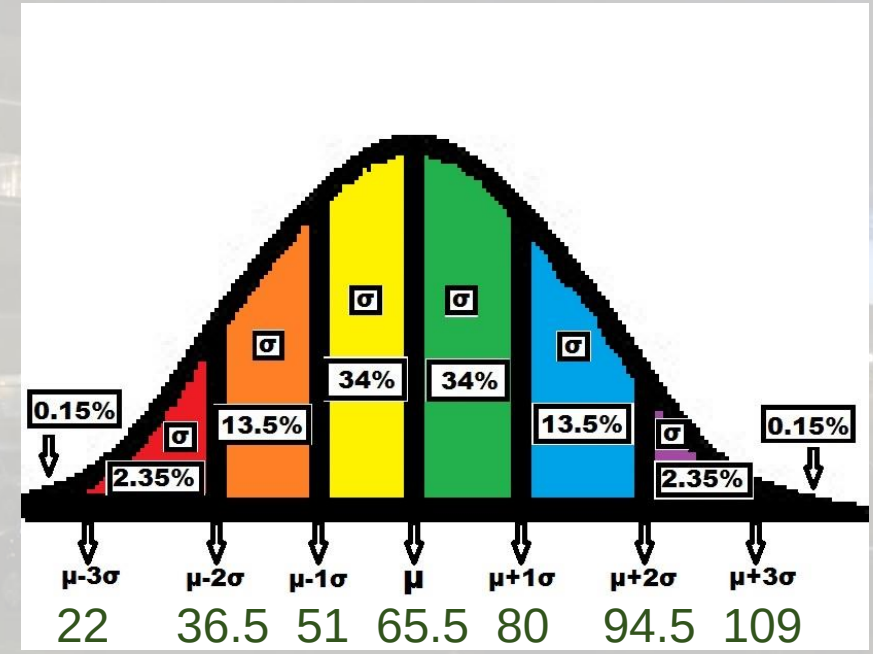
Answer: 0.15% + 2.35% + 13.5% = 16%. Top 16% cutpoint is 80.

Applying Empirical Rule



Question: What % of scores are below 94.5?

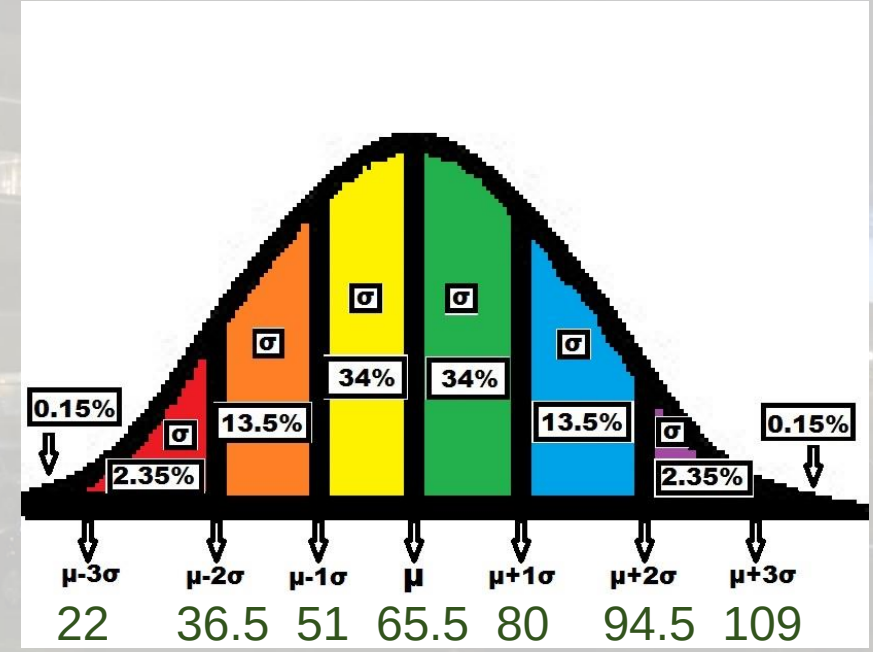
Applying Empirical Rule



Question: What % of scores are below 94.5?

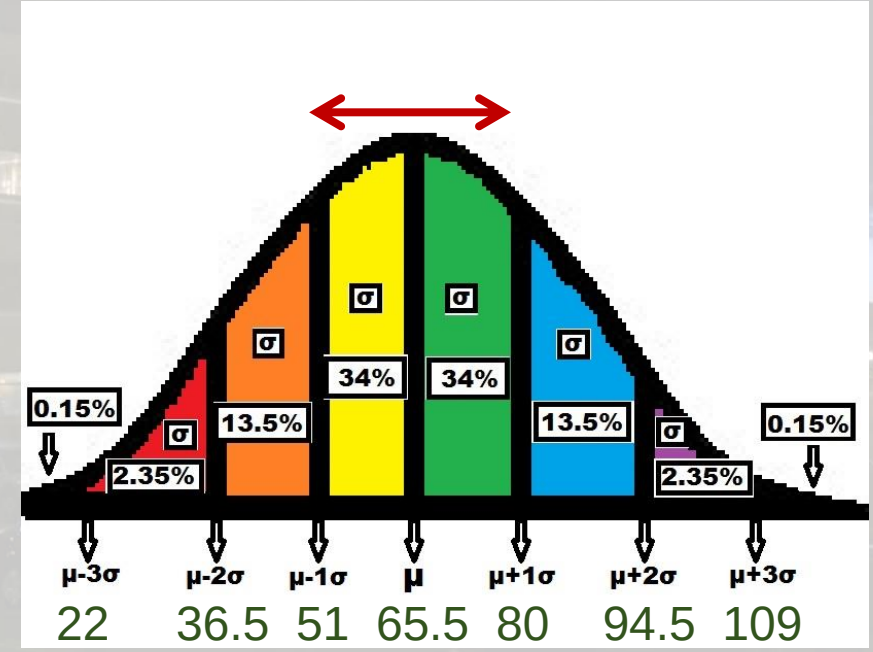
Answer: Add up % below 94.5: 13.5% + 34% + 34% + 13.5% + 2.35% + 0.15% = 97.5%

Applying Empirical Rule



Question: What are the cutpoints for the middle 68% of the data?

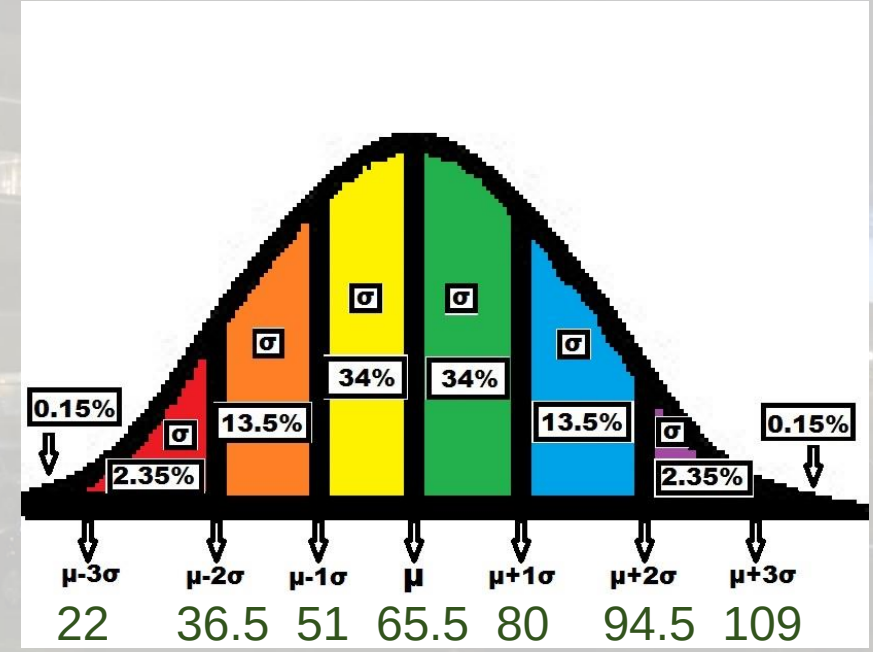
Applying Empirical Rule



Question: What are the cutpoints for the middle 68% of the data?

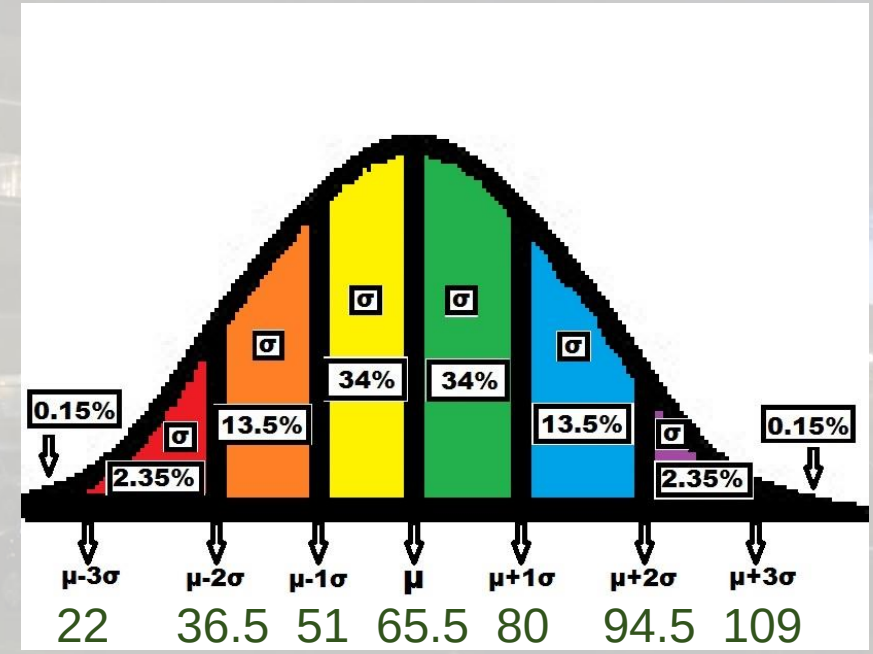
Answer: Middle 68% means 34% above mean (80) and 34% below mean (51).

Applying Empirical Rule



Question: What is the *probability* that if I select one student, that student will have a score less than 80?

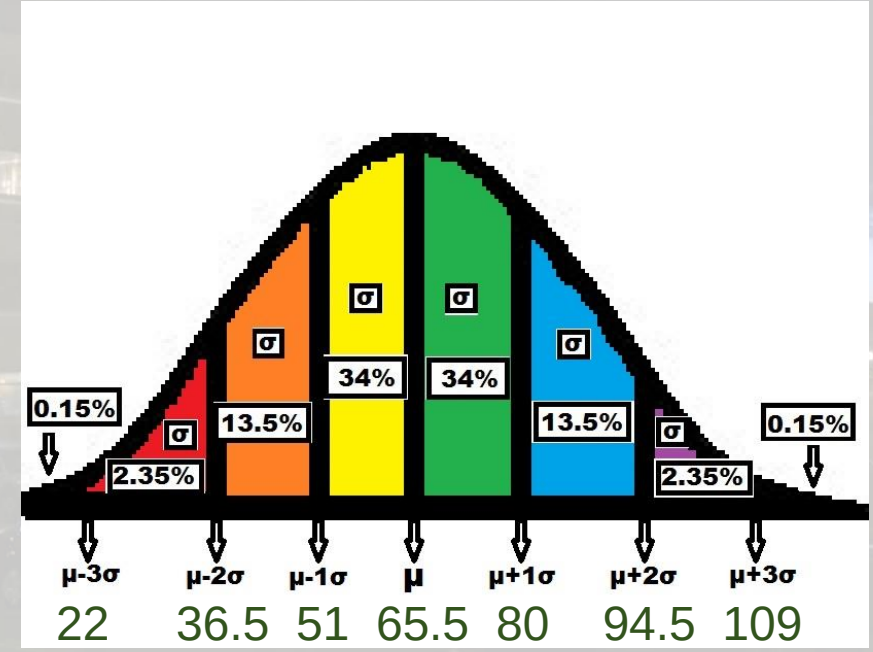
Applying Empirical Rule



Question: What is the *probability* that if I select one student, that student will have a score less than 80?

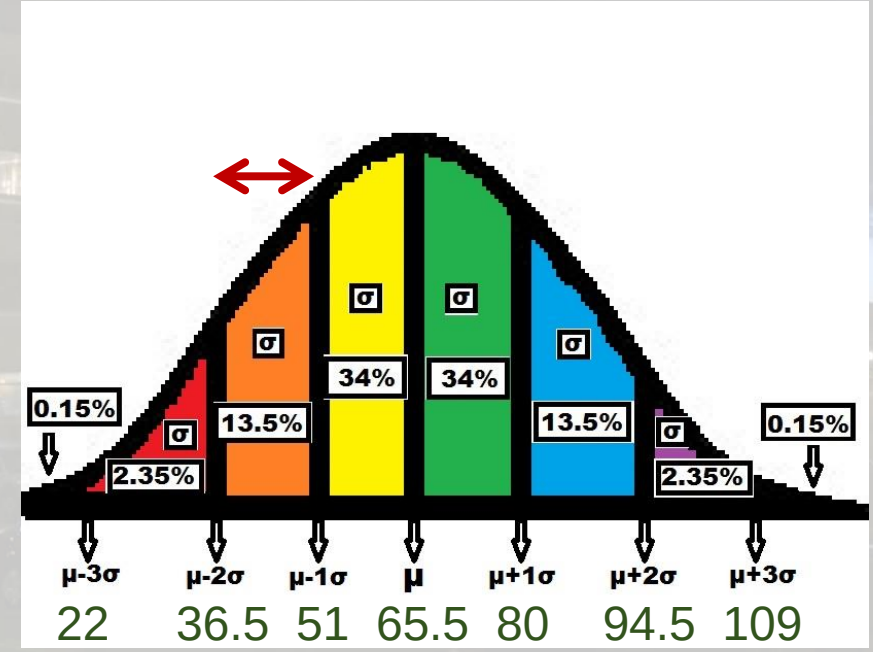
Answer: 50% + 34% = 84%

Applying Empirical Rule



Question: What is the *probability* I will select a student with a score between 36.5 and 51?

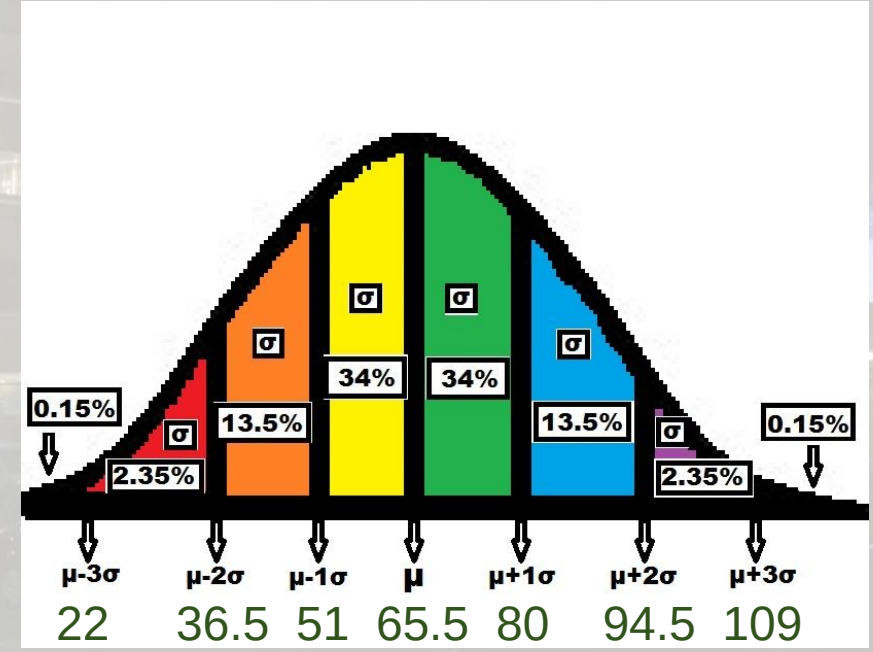
Applying Empirical Rule



Question: What is the *probability* I will select a student with a score between 36.5 and 51?

Answer: 13.5%

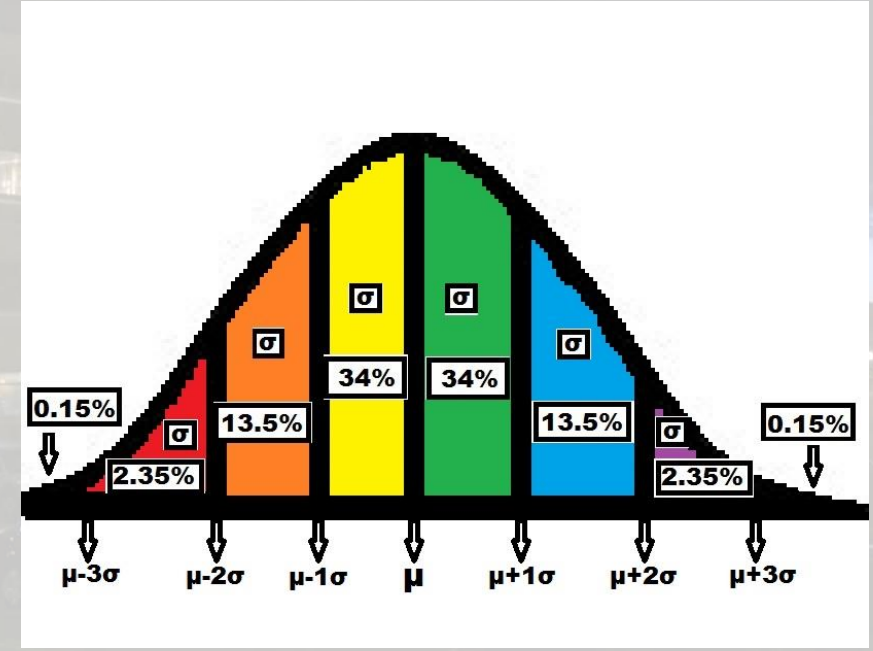
Applying Empirical Rule



Think about what would happen in a different class taking the same hard test.

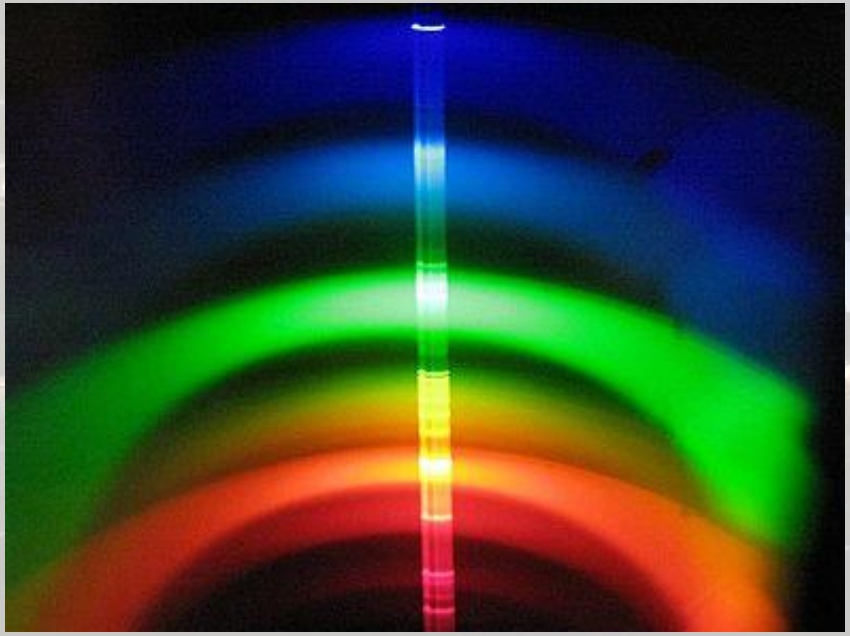
- What if the μ was the same, but the σ was larger than 14.5? What would that do to the intervals?
- What if σ was smaller than 14.5?

%, Area, and Probability: Did you Know?



- The %'s literally refer to the % of the area of the shape.
- Imagine the whole thing as 100%.
- Example:
  - The orange part is 13.5% of the area
  - It is also the probability that an "x" between μ-1σ and μ-2σ will be selected

Conclusion



- The Empirical Rule helps establish intervals that apply to normally distributed data
- These intervals have a certain percentage of the data points in them
- These intervals depend on the mean and standard deviation of the data distribution

Image by Zaereth